Query Optimization for Differentially Private Data Management Systems

Shangfu Peng[1], Yin Yang[2], Zhenjie Zhang[2], Marianne Winslett[2,3], Yong Yu[4]

[1]*Department of Computer Science, University of Maryland at College Park, MD, USA*
shangfu@cs.umd.edu

[2]*Advanced Digital Sciences Center, Singapore*
{yin.yang, zhenjie}@adsc.com.sg

[3]*Department of Computer Science, University of Illinois at Urbana-Champaign, IL, USA*
winslett@illinois.edu

[4]*Department of Computer Science and Engineering, Shanghai Jiao Tong University, China*
yyu@cs.sjtu.edu.cn

Abstract— Differential privacy (DP) enables publishing the results of statistical queries over sensitive data, with rigorous privacy guarantees, and very conservative assumptions about the adversary's background knowledge. This paper focuses on the *interactive* DP framework, which processes incoming queries on the fly, each of which consumes a portion of the user-specified *privacy budget*. Existing systems process each query independently, which often leads to considerable privacy budget waste and consequently fast exhaustion of the total budget. Motivated by this, we propose *Pioneer*, a query optimizer for an interactive, DP-compliant DBMS. For each new query, Pioneer creates an *execution plan* that combines past query results and new results from the underlying data. When a query has multiple semantically equivalent plans, Pioneer automatically selects one with minimal privacy budget consumption. Extensive experiments confirm that Pioneer achieves significant savings of the privacy budget, and can answer many more queries than existing systems for a fixed total budget, with comparable result accuracy.

I. INTRODUCTION

Differential privacy [4] (DP) is one of the strongest schemes for protecting individuals' privacy in published group statistics. DP ensures it is provably hard for an adversary to infer the presence or absence of any specific record, even if the adversary knows all other DB records. DP has rapidly gained popularity in multiple research fields, including computer networking [19], data mining [8], and medical analysis [30]. A common solution for DP is to inject random noise into statistical query results. The noise scale is controlled by a parameter ϵ called the *privacy budget*. A smaller ϵ leads to more noise and stronger privacy in the sense of increased difficulty for the adversary to derive sensitive information.

DP can be *offline* or *interactive*. The former answers a pre-defined set of queries and publishes just those answers, while the latter processes incoming queries as they arrive. We focus on the latter, which enables *DP-compliant DBMSs* with a broad range of applications. One example is to build such a DB for electronic health records, which answers queries about the number of patients suited for a clinical trial. A pharmaceutical company trial needs patients with particular medical conditions and demographics. Before committing to a trial in, say, Singapore, the company must determine whether Singapore has enough patients satisfying the criteria. Since different trials have different patient criteria, the patient count queries cannot be computed in advance. A DP-compliant DBMS fits well here, as it can answer incoming patient count queries while protecting individuals' privacy.

Figure 1 illustrates the architecture of a DP-compliant DBMS, in which the client issues every statistical query q to a query executor. The latter then computes the exact result of q over the DB, and returns a perturbed version of the result to the client. Because different queries may have different accuracy requirements (e.g., a patient count query for a rare disease might need to be answered more accurately than that for a common disease), the client also specifies the required accuracy of q, e.g., in terms of variance[1]. To satisfy DP, the DBMS maintains a total privacy budget ϵ_{tot}, whose initial value is set by the administrator. As we explain in Section II-A, the execution of each query q consumes a portion of the privacy budget ϵ_q, which is deducted from ϵ_{tot}. If $\epsilon_q > \epsilon_{tot}$, q is rejected; when ϵ_{tot} decreases to zero, no more queries can be answered. Clearly, minimizing ϵ_q is critical for the DBMS.

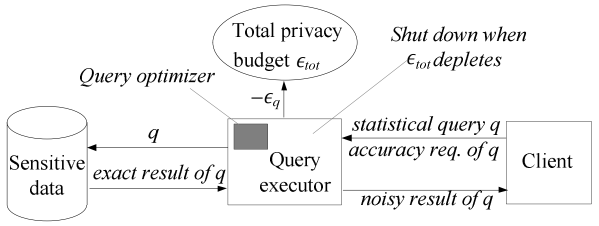


Figure 1: Architecture of a DP-compliant DBMS

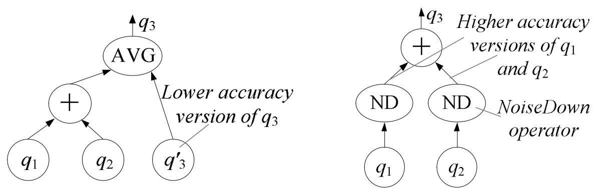
Existing DP-compliant DBMSs, e.g., PINQ [16], add noise to the results of each query independently. We observe that often,

[1] Some existing systems require the client to specify the privacy budget of each query, rather than its accuracy. Our discussions with potential users reveal that it is hard for them to determine the appropriate privacy budget, and specifying the required accuracy is much more intuitive. Similar observations are made in [21].

significant privacy budget can be saved by reusing previous query results to answer a new query. Consider queries q_1-q_3 in Figure 2a, where the exact answer to q_3 is the sum of those of q_1 and q_2, and the DBMS has already processed q_1 and q_2. Suppose that the noisy answers to q_1 and q_2 are R_1 and R_2, with variance v_1=2 and v_2=2, respectively, according to the client's requirements. Since the noise injected into each result has mean value 0 [6], R_1+R_2 has variance 2+2=4 with respect to the exact answer of q_1+q_2. Hence, if the requested variance for q_3 is v_3=4, we can simply answer q_3 by R_1+R_2 without spending any privacy budget at all.

q_1: SELECT count(*) FROM Patients WHERE age>=20 AND age <25
q_2: SELECT count(*) FROM Patients WHERE age>=25 AND age <30
q_3: SELECT count(*) FROM Patients WHERE age>=20 AND age <30

(a) Three correlated queries


q_3
AVG
+
Lower accuracy version of q_3
q_1
q_2
q'_3
q_3
+
Higher accuracy versions of q_1 and q_2
ND
ND
NoiseDown operator
q_1
q_2

(b) Execution plan EP_1 for q_3 (c) Execution plan EP_2 for q_3
Figure 2: Semantically equivalent plans for incoming query q_3

Now consider a more complicated case with the same queries, and accuracy requirements v_1=v_2=v_3=2. R_1+R_2 no longer answers q_3 with sufficient accuracy, and the query executor must retrieve new information from the sensitive data. On the other hand, reusing R_1 and R_2 can still reduce the privacy budget for answering q_3. Figure 2b shows an execution plan EP_1 for q_3 that first adds R_1 and R_2, and then averages R_1+R_2 with the noisy answer of another query q'_3, which is identical to q_3 except that it has a lower accuracy requirement v'_3=4. The result has variance (4+4)/2^2=2, which satisfies q_3. As we explain in Section II-A, the privacy budget consumed by a query is inversely proportional to the square root of its variance. This means that the privacy cost of q'_3 is only $\sqrt{1/2}$ of that of q_3. Since q'_3 is the only operator that requires sensitive information, the total privacy cost of plan EP_1 is significantly smaller than for computing q_3 directly.

Figure 2c shows another plan EP_2 for q_3, which involves the *NoiseDown* (*ND*) operator [25]. ND takes the noisy result of a previous query (e.g., q_1 with v_1=2), and outputs a more accurate version (e.g., with v'_1=1). The additional privacy cost of ND is the difference between the costs of answering the same query with the two different values for noise variance. In our running example with v_1=v_2=v_3=2, EP_2 refines R_1 and R_2 to obtain their answers with variance v'_1=v'_2=1, and adds them to answer q_3 with variance v'_1+v'_2=2=v_3. As we explain in Section III, the privacy cost of EP_2 in this particular case is higher than for EP_1; the reverse is true in other settings, e.g., if v_1=1, v_2=v_3=2. This means that the relative performance of different plans depends on the workload.

There are two main challenges in realizing an effective optimizer for DP-compliant DBMSs. First, there is a vast search space for possible execution plans for answering an incoming query q, and it is nontrivial to select the best plan. The example in Figure 2, for instance, involves only two past queries and two simple plan types; yet the optimizer must determine several parameters (i.e., v'_3 in EP_1, v'_1 and v'_2 in EP_2), and select the plan that minimizes privacy budget usage. As we show in the paper, more general and effective plans are also more complex, requiring sophisticated algorithms for plan building and selection. The second challenge lies in finding past query sets that can be used to build a plan for the new query q. There can be a large number of past queries with numerous possible combinations; meanwhile, an obvious combination of past queries that can answer q may not exist. For example, in Figure 2, if we modify q_1's selection condition to "age between 10 and 15", q_3=q_1+q_2 no longer holds, and there is no apparent way to answer q_3 with q_1 and q_2. We face these challenges with a novel solution *Pioneer*, an effective and generic query optimizer for DP-complaint DBMSs. Pioneer selects the best plan based on rigorous analysis of various plan types, and creates opportunities for re-using past queries by exploiting unique properties of query processing under DP. Further, Pioneer incurs negligible computation costs. For simplicity, we present Pioneer for linear counting queries [14], a fundamental task in DP. Nevertheless, Pioneer applies to a wider class of queries that satisfy its assumptions. Extensive experimental studies demonstrate that Pioneer achieves significant privacy budget savings, and thus can answer significantly more queries than existing methods.

In the following, Section II reviews related work, Sections III and IV present the plan building and past query selection modules, respectively. Section V presents a thorough experimental evaluation. Section VI concludes the paper.

II. RELATED WORK

Section II-A provides necessary background on DP. Section II-B overviews query processing techniques in DP systems.

A. *Differential Privacy Preliminaries*

Recall from Section I that DP guarantees it is hard for the adversary to infer the presence or absence of any individual record, even if she knows the details of all remaining records. DP achieves this by requiring the (noisy) query results to follow similar probability distributions for every two *neighbor databases* DB_1 and DB_2. DB_1, DB_2 are neighbors iff DB_2 can be obtained by adding or removing one record to/from DB_1. Let q be an aggregate query, s a possible noisy answer to q, and $q(DB)$ a random variable representing the answer given to the user. The most commonly used version of DP, called ϵ-*differential privacy* (ϵ-*DP*) [4], requires the following:

$$Prob(q(DB_1) = s) \leq e^{\epsilon} \cdot Prob(q(DB_2) = s). \qquad (1)$$

Intuitively, ϵ-DP ensures that the change in probability caused by adding/removing a record is bounded by a constant factor e^{ϵ}. ϵ-DP is *composable* [4], meaning that a system satisfies $(\sum_{i=1}^{m} \epsilon_i)$-DP after answering queries q_1, …, q_m independently with ϵ_1-, …, ϵ_m-DP guarantees, respectively. This property is the foundation for an interactive DP-compliant database, in which each query consumes a portion of the total privacy budget ϵ_{tot}. After ϵ_{tot} depletes and the database is shut down, composability guarantees that the system satisfies ϵ_{tot}-DP.

The Laplace mechanism [6] offers an effective way to achieve ϵ-DP, by adding random Laplace noise to the exact query answers. The noise has mean 0, and a scale of Δ_q/ϵ, where Δ_q is the *sensitivity* of the query q, defined as the maximum possible difference in the actual results of q on any two neighbor databases. Formally, we have:

$$\Delta_q = \max_{DB_1, DB_2} |q^{ACT}(DB_1) - q^{ACT}(DB_2)|, \qquad (2)$$

where $q^{ACT}(DB)$ denotes the exact answer to q over DB, and DB_1, DB_2 are two arbitrary neighbor databases. For instance, the sensitivity of a selection-count query is 1, since adding / removing a record can affect the query result by at most 1. Accordingly, the injected Laplace noise has a scale of $1/\epsilon$.

Because the Laplace mechanism injects zero-mean noise into the query results, the variance of the noisy result is the main measure of its error. Let v_i be the variance of the noisy results for q_i, ϵ_i its privacy budget consumption, and Δ_i its sensitivity. Properties of the Laplace distribution ensure:

$$v_i = 2 \cdot (\Delta_i/\epsilon_i)^2, \qquad \epsilon_i = \sqrt{2/v_i} \cdot \Delta_i. \qquad (3)$$

Equation (3) shows that the budget consumption for a query is inversely proportional to the square root of its required variance. In Figure 2, plan EP_1 uses this to reduce the budget cost of q_3, by processing a lower-accuracy version q'_3 of q_3.

The exponential mechanism [20] is another popular method to enforce ϵ-DP, e.g., on queries with categorical results. Because certain queries have high sensitivity, previous work has proposed relaxations of ϵ-DP, e.g., (ϵ,δ)-DP [5], which guarantees ϵ-DP with a certain probability. Mechanisms that comply with ϵ-DP also satisfy (ϵ,δ)-DP, but the reverse is usually not true, e.g., methods proposed in [14][17] satisfy (ϵ,δ)-DP, but not ϵ-DP. This paper focuses on the stronger ϵ-DP definition, though some ideas also apply to (ϵ,δ)-DP.

B. *Differentially Private Query Processing*

Although the Laplace mechanism can answer all types of queries under DP, straightforward applications of this method to practical query workloads often lead to poor result accuracy with given values for ϵ, or, in our setting with user-specified error levels, considerable waste of privacy budget. Therefore, researchers have proposed numerous algorithms suitable for specific applications. This section focuses on query processing techniques proposed by the database community [28].

Offline methods. Xiao et al. [26] investigate range-count queries, and propose an offline solution *Privlet*, which takes ϵ as input and publishes the results of all unit-range counts. The answer to a query q with a wider range is derived by adding the unit-size counts within q's range. Privlet guarantees that the noise variance of any one-dimensional range count query is bounded by $O(\log^3 n/\epsilon^2)$, where n is the domain size on which the range selection is performed. Hay et al. [10] tackle the problem that the noisy answers to different queries are often inconsistent [1], and design effective methods to output consistent (and more accurate) results. Refs. [3][22] propose synopsis for multi-dimensional range counts. Chen et al. [2] solve the problem of publishing set-valued data under DP. Ding et al. [7] study optimal data cube publication under DP. Xu et al. [27] and Li et al. [15] use compression techniques to reduce the noise in a differentially private synopsis.

The main limitation of non-interactive methods is that their privacy budget usage is determined by the highest possible required accuracy for a query. Consequently, they are not suitable for applications with very different accuracy requirement for different queries. In practice, an application may only need to answer a small number of queries with high accuracy (e.g., those critical to a clinical trial), and the remaining ones with low accuracy. In an extreme case, even if only one query needs to be answered with high accuracy v_{high}, a non-interactive method still needs to build a synopsis whose accuracy lower bound exceeds v_{high}, leading to tremendous waste in privacy budget. Interactive methods, reviewed next, handle such situations much more effectively.

Interactive methods. Existing interactive DP systems, e.g., PINQ [16], answer each incoming query independently, without reusing previous query results. Li et al. [13][14] investigate the processing of a given batch of linear counting queries, and propose methods that process them more efficiently than answering them individually. Queries in different batches are still processed independently. Yuan et al. [29] targets the same problem, and propose improved solutions. Rastogi and Nath [23] study the processing DP queries over time series DBs, where queries have very high sensitivity. Machanavajjhala et al. [18] apply DP to social recommendation systems. These methods can be used in combination with the proposed approach. Interactive DP work in the theory community (e.g., [9][24]) does not discuss the opportunity of reusing existing query results, either.

Xiao et al. [25] develop techniques that minimize the total relative error of a given batch of queries. The contributions of [25] include the ND operator mentioned in Section I. Specifically, suppose that query q_i has been answered with a noise variance of v_i; the ND operator refines the accuracy of q_i to a new variance $v'_i < v_i$, with a privacy budget as follows.

$$\epsilon_i^{ND} = \left(\sqrt{2/v'_i} - \sqrt{2/v_i}\right) \cdot \Delta_i. \qquad (4)$$

The ND operator does not waste any budget, because the privacy cost of first answering q_i with variance v_i, and then refining it to variance v'_i with ND is the same as answering q_i directly with variance v'_i. ND achieves this by adding noise with a complicated distribution that depends on both the actual result of q, and the previous noisy result with variance v_i. The method of [25] relies on unique properties of the Laplace distribution; consequently, ND cannot refine a composite result, e.g., the sum of multiple noisy results, which no longer follow the Laplace distribution. For this reason, In Figure 2, one cannot put ND after the sum operator. Finally, Kifer and Machanavajjhala [11][12] discuss the limitations of DP in social network publication, suggesting that an even stronger privacy definition might be needed for certain applications.

III. Building An Execution Plan

This section describes how Pioneer constructs the best query plan for an incoming query q given a set of relevant queries Q.

Pioneer requires (i) all past queries operate on the same dataset, and contain the same aggregate clause, e.g., COUNT, SUM, etc. as the new query; (ii) noise added to each query in Q follow zero-mean, Laplace distributions. Besides these, Pioneer does not assume any specific query type. For simplicity, we use linear counting queries in our presentation, and explain the difference in processing other query types whenever necessary. As in most existing work on DP, Pioneer measures the accuracy of a query result by its noise variance.

Section IV explains how to find Q (i.e., answerability of a new query with past queries) and divide it into subsets $Q=\cup_{1\le i\le n} Q_i$, such that for each Q_i, the exact answer to q is a linear combination of the exact answers to the queries in Q_i, for every DB instance. In our running example in Figure 2, we have $q=q_3$, and $Q=Q_1=\{q_1, q_2\}$, i.e., the linear combination q_1+q_2 is equivalent to q. More formally, if Q_i contains queries $q_{i,1}$, …, q_{i,m_i}, then there must be a constant $c_{i,j}$ associated with each query $q_{i,j}$, such that:

$$q = \sum_{j=1}^{m_i} (c_{i,j}.q_{i,j}), \forall 1 \le i \le n. \tag{5}$$

Note that $c_{i,j}$ can be *negative*, e.g., $q=q_{1,1}+q_{1,2}-q_{1,3}$. As explained in Section IV, negative coefficients greatly increase the flexibility for composing q with past queries. To avoid notational overload, we set all coefficients $c_{i,j}$ to 1 until the end of Section III-C:

$$q = \sum_{j=1}^{m_i} q_{i,j}, \forall 1 \le i \le n. \tag{6}$$

Until the end of Section III-C, we consider only queries with sensitivity 1, e.g., count queries, as explained in Section II-A. Until Section III-D, we require that each query occur in at most one subset Q_i of Q.

In the following, Sections III-A to III-D presents 4 types of query plans: *top branching* (*TB*) *plans*, *leaf refining* (*LR*) *plans*, TB-LR hybrid plans, and composite plans. Among these, TB and LR plans show the basic idea of reusing results from past queries. For instance, plans EP_1 and EP_2 in Figure 2 are example TB and LR plans, respectively. The hybrid plan generalizes these two basic plan types, and performs at least as good as the best TB and LR plan. Finally, the composite plan further generalizes the hybrid plan to handle overlapping query sets in Q. The plan creation module of Pioneer is based on the algorithm for composite plans, which subsumes the other three simpler plan types. Table I summarizes symbols used throughout the paper.

Table I
LIST OF COMMON NOTATION

Symbol	Meaning
q, v	Incoming query and its specified noise variance
Q	Set of relevant previous queries
Q_i, m_i	i-th partition of Q and the number of queries therein
n	Number of partitions of Q
$q_{i,j}$, $v_{i,j}$	A query in Q_i and its noise variance
w_i, w'_i	Weight associated with partition Q_i in the TB and LR plans
q', v'	New query processed in a TB plan, and its noise variance
$v'_{i,j}$	Refined noise variance of query $q_{i,j}$ in an LR plan
ϵ_{plan}	Privacy budget consumption of a plan, e.g., TB, LR, hybrid

A. TB Plans

Figure 3 illustrates the structure of a TB plan. First, for each partition Q_i of Q, the plan computes the sum of Q_i's results, using Equation (6). Additionally, the TB plan answers $q'=q$, but with a lower accuracy v' than the required accuracy v for q. These answers are fed into a single weighted average operator AVG, whose output answers q with sufficient accuracy.

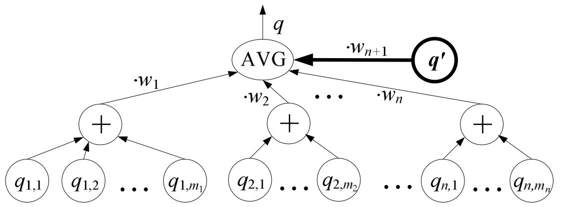


Figure 3: Structure of a TB plan

The privacy budget consumption of the TB plan equals to that of answering the query q', which, in turn, is determined by the required accuracy v' of q'. Hence, in the following we derive the appropriate plan parameters to minimize v'. These parameters are the weights assigned to the n+1 inputs of the AVG operator. Suppose that all these weights sum up to 1. Let w_i ($1\le i\le n$) denote the weight assigned to the output of the i-th sum operator. Then, the weight assigned to q' is $1-\sum_{i=1}^{n} w_i$.

Let $v_{i,j}$ be the noise variance of query $q_{i,j}$, and $v_{i,*}$ be the variance for the output of the i-th sum that covers query set Q_i. Since the noise for each query follows an independent, zero-mean Laplace distribution, $v_{i,*}$ is simply the sum of variance for queries in Q_i, i.e., $\sum_{j=1}^{m_i} v_{i,j}$. Therefore, we have:

$$v = \sum_{i=1}^{n} w_i^2 \cdot v_{i,*} + \left(1 - \sum_{i=1}^{n} w_i\right)^2 \cdot v'. \tag{7}$$

The minimum value of v' is obtained, when the partial derivative of Equation (7) with respect to each w_i equals 0. Accordingly, we solve for the optimal values for w_i and v', and obtain the following:

$$w_i = \frac{v}{v_{i,*}} (1 \le i \le n), \qquad v' = \frac{1}{\frac{1}{v} - \sum_{i=1}^{n} \frac{1}{v_{i,*}}} \tag{8}$$

If the denominator in Equation (8) yields a non-positive value, then q' is unnecessary, and the rest of the plan answers q with sufficient accuracy. Finally, combining Equation (8) and our simplifying assumption that each query has sensitivity 1 (which is removed in Section III-C), we obtain privacy budget cost of the TB plan, as follows.

$$\epsilon_{TB} = \sqrt{max\left(0, \frac{2}{v} - \sum_{i=1}^{n} \frac{2}{v_{i,*}}\right)}, \tag{9}$$

In the running example in Figure 2, according to the above analysis, the privacy cost for plan EP_1 for the setting $v_1=v_2=v_3=2$ is $\sqrt{1/2}$, which is reached when $v'_3=4$. Similarly, for the setting $v_1=1$, $v_2=v_3=2$, the minimum budget consumption is $\sqrt{1/3}$, with $v'_3=6$. In both cases, the privacy cost is significantly lower than that of answering q directly, i.e., 1 according to Equation (3).

B. LR Plans

Figure 4 shows the LR plan, which uses the NoiseDown (ND) operator described in Section II-B, as well as the sum and AVG operators seen earlier. The accuracy of each previous query result $q_{i,j}$ is refined with an ND, and then fed into its corresponding sum. Note that as explained in Section II-B, one cannot place an ND downstream of the sum or AVG operators.

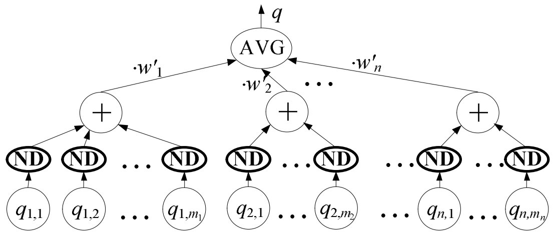


Figure 4: Structure of an LR plan

The LR plan consumes privacy budget through the ND operators. Hence, in the following we focus on minimizing the sum of privacy cost of all NDs in the plan. According to Equation (4), the budget cost of an ND is determined by the noise variance of its input and output. Let $v'_{i,j}$ (referred to as the *refined variance*) be the variance of the output of the ND for query $q_{i,j}$. Similar to the TB plan, we define $v'_{i,*}=\sum_{j=1}^{m_i} v'_{i,j}$, and use w'_i be the normalized weight assigned to the i-th input to AVG. Similar to the analysis in Section III-A, the best weights are as follows.

$$w'_i = v/v'_{i,*}\,(1 \le i \le n), \qquad v = 1\Big/\sum_{i=1}^{n}\frac{1}{v'_{i,*}}. \tag{10}$$

By Equation (4), for queries with sensitivity 1, the total privacy cost of the LR plan is:

$$\epsilon_{LR} = \sum_{i=1}^{n}\sum_{j=1}^{m_i}\left(\sqrt{2/v'_{i,j}} - \sqrt{2/v_{i,j}}\right). \tag{11}$$

The first derivative of ϵ_{LR} with respect to variance $v'_{i,j}$ is then:

$$\frac{\partial \varepsilon_{LR}}{\partial v'_{i,j}} = \frac{-1}{v'_{i,j}\cdot\left(2v'_{i,j}\right)^{1/2}}, \forall 1 \le i \le n, 1 \le j \le m_i. \tag{12}$$

Equation (12) suggests that the *marginal privacy cost* with respect to $v'_{i,j}$ grows as $v'_{i,j}$ decreases. Intuitively, this means we give up more privacy to refine a query result that is already quite accurate, than to refine another that is less accurate. Meanwhile, reducing the variance of any of queries in the same partition Q_i by a constant δ has the same effect on the plan, i.e., decreasing $v_{i,*}$ by δ. Without loss of generality, we assume that queries in Q_i are sorted in ascending order of their original accuracy, i.e., $v_{i,j} \ge v_{i,j+1}$, for all i and j. It follows that the best LR plan should first refine the most inaccurate query in Q_i, i.e., $q_{i,1}$, to the level of the second most inaccurate query, i.e., $v_{i,2}$, and then reduce both of them to $v_{i,3}$, and so on. These observations are summarized in the following lemma.

Lemma 1: Suppose that we are given the value for $v'_{i,*}$ in each partition Q_i of the best LR plan, and $v_{i,j} \ge v_{i,j+1}$, for all $1 \le i \le n$, $1 \le j < m_i$. If $v'_{i,*} \le m_i \cdot v_{i,m_i}$, then the best LR plan satisfies $v'_{i,j}=v'_{i,*}/m_i$ for $1 \le j < m_i$. Otherwise, there exists $1 \le l < m_i$ such that:

$$\sum_{j_1=1}^{l-1} v_{i,l} + \sum_{j_2=l}^{m_i} v_{i,j_2} \le v'_{i,*} < \sum_{j_3=1}^{l} v_{i,l+1} + \sum_{j_4=l+1}^{m_i} v_{i,j_4}.$$

Further, the best LR plan satisfies $v'_{i,j}=v_{i,l}$, for $1 \le j < l$, and $v'_{i,j}=v_{i,j}$, for $l \le j \le m_i$.

Next we focus on the computation of the best values for $v'_{i,*}$ $(1 \le i < n)$. Consider v as a function of $v'_{i,*}$ $(1 \le i < n)$. By Equation (10), the first derivative of v with respect to each $v'_{i,*}$ reflects the *marginal accuracy benefit* of reducing $v'_{i,*}$, which is:

$$\frac{\partial v}{\partial v'_{i,*}} = \frac{\partial v}{\partial v'_{i,j}} = \frac{v^2}{\left(v'_{i,*}\right)^2}, \forall 1 \le i \le n, 1 \le j \le m_i. \tag{13}$$

This equation implies that the marginal accuracy benefit of reducing $v'_{i,*}$ increases as $v'_{i,*}$ decreases. But as discussed above, the marginal privacy cost for reducing $v'_{i,*}$ also increases as $v'_{i,*}$ decreases. This conflict makes finding the best LR plan rather complicated. We employ a heuristic approach, defining the *score* of a query $q_{i,j}$ as the ratio of its marginal accuracy benefit and marginal privacy cost:

$$score_{i,j} = -\frac{\partial v/\partial v'_{i,j}}{\partial \epsilon_{LR}/\partial v'_{i,j}} = \frac{v^2\cdot v'_{i,j}\cdot\left(2v'_{i,j}\right)^{1/2}}{\left(v'_{i,*}\right)^2}. \tag{14}$$

Figure 5 gives our *CLRP* algorithm for building an LR plan. CLRP initializes the refined variance $v'_{i,j}$ of each query $q_{i,j}$ to the original variance $v_{i,j}$ (line 2), and calculates its score accordingly. Then CLRP iterates (lines 4-16), preserving the condition in Lemma 1 at all times. Each iteration starts by selecting a partition Q_i that contains queries with the highest score (line 5). The marginal accuracy benefit defined in Equation (13) is the same for all queries in Q_i; the query $q_{i,1} \in Q_i$ has the lowest marginal privacy cost, by Lemma 2. After that, CLRP attempts to refine the accuracy of $q_{i,1}$ and all other queries $q_{i,2}$-$q_{i,j}$ in Q_i with the same refined accuracy as $q_{i,1}$. The target is to reduce their variance to $v'_{i,j+1}$, i.e., the query with accuracy just above $q_{i,1}$-$q_{i,j}$ in Q_i (line 11). If such refinement leads to a higher output accuracy than required for q, CLRP instead computes $v'_{i,1}$-$v'_{i,j}$ using v and Lemma 2 (lines 6-9 & 12-14). Finally, CLRP updates $v'_{i,*}$ and the scores of refined queries, then goes to the next iteration. CLRP ends when the LR plan's output satisfies accuracy requirement v.

Regarding computational overhead, each CLRP iteration takes constant time. The number of iterations is bounded by the number of queries in Q, since in every iteration but the last, the algorithm selects a group of queries with the same refined variance, and decreases their variance to the original variance of a target query (line 11); further, the target query is different in each iteration. Therefore, the running time of CLRP is linear in the number of past queries in Q. In Figure 2, EP_2 is an LR plan. For the setting $v_1=v_2=v_3=2$, CLRP returns $v'_1=v'_2=1$, using privacy budget $2\cdot(\sqrt{2}-1)\approx 0.83$, which is higher than that of EP_1 ($\sqrt{1/2}\approx 0.71$). On the other hand, for the setting $v_1=1$, $v_2=v_3=2$, the budget consumption of EP_2 is $\sqrt{2}-1\approx 0.41$, reached when $v'_1=v'_2=1$, which is lower than the privacy budget cost of EP_1 ($\sqrt{1/3}\approx 0.57$). These facts confirm the need for an effective query optimizer.

```
*CLRP*($Q$, $v_{i,j}$ for all $q_{i,j}$, $q$, $v$) // Create the LR plan
// Input: $Q$, $v_{i,j}$: set of previous queries and their result noise variance
//        $q$, $v$: incoming query and its required result variance
// Output: $v'_{i,j}$: the refined noise variance for each query in $Q$
1.  For each pair of $i,j$, $1 \le i \le n$, $1 \le j < m_i$
2.      Initialize $v'_{i,j} = v_{i,j}$
3.      Calculate $score_{i,j}$ according to Equation (14)
4.  Repeat
5.      Select a partition $Q_i$ that maximizes $score_{i,1}$
6.      If all queries in $Q_i$ have the same refined variance
7.          Solve $v'_{i,*}$ from Equation (10)
8.          Set the refined variance of all queries in $Q_i$ to $v'_{i,*}/m_i$
9.          Goto line 17 // break the repeat-until loop
10.     Let $q_{i,j}$ be the query with maximum $j$ satisfying $v'_{i,j} = v'_{i,1}$
11.     Refine the variance of queries $q_{i,1}$-$q_{i,j}$ to $v'_{i,j+1}$
12.     If the output of the LR plan has variance smaller than $v$
13.         Roll back the refinement step in Line 11
14.         Solve $v'_{i,*}$ from Equation (10), and set the refined
        variance of $q_{i,1}$-$q_{i,j}$ according to Lemma 2
15.     Update the score for each query in $Q_i$
16. Until Equation (10) is satisfied
17. Return $v'_{i,j}$ for all $1 \le i \le n$, $1 \le j < m_i$
```

Figure 5: CLRP algorithm: Create an LR plan

C. Hybrid Plans

As shown in Figure 6, the hybrid plan integrates the TB plan idea of issuing a lower-accuracy version of q, plus the LR plan idea of refining past queries with the ND operator. Specifically, let $v_{i,j}$ and $v'_{i,j}$ be the original and refined noise variance of query $q_{i,j}$, respectively, and v, v', $v'_{i,*}$ be the noise variance of q, q', and the output of the i-th sum operator, respectively. Combining the analysis of the TB and LR plans, we have:

$$v = \sum_{i=1}^{n} \left((w'_i)^2 \cdot v'_{i,*} \right) + \left(1 - \sum_{i=1}^{n} w'_i \right)^2 \cdot v'. \qquad (15)$$

where the best value for the weight w'_i is $w'_i = v/v'_{i,*}$ ($1 \le i \le n$).

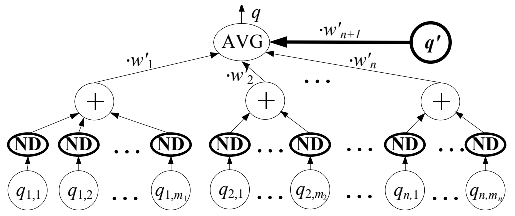


Figure 6: Structure of a hybrid plan

Let ϵ_{hybrid} denote the total privacy cost of the hybrid plan:

$$\epsilon_{hybrid} = \sqrt{2/v'} + \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left(\sqrt{2/v'_{i,j}} - \sqrt{2/v_{i,j}} \right). \qquad (16)$$

Since the hybrid plan essentially outputs the weighted average of new query q' and an LR sub-plan, Lemma 2 also holds. The hybrid plan creation algorithm *CHP*, shown in Figure 7 resembles the CLRP algorithm (Figure 5) for creating the LR plan. Specifically, similar to the case of LR plans, we define the score for each $v'_{i,j}$, $1 \le i \le n$, $1 \le j \le m_i$, as follows.

$$score_{i,j} = \frac{v'_{i,j} \cdot \left(v'_{i,j} \right)^{1/2}}{\left(v'_{i,*} \right)^2} \propto \frac{-\partial v / \partial v'_{i,j}}{\partial \epsilon_{hybrid} / \partial v'_{i,j}} \qquad (17)$$

CHP initializes and updates the values for the $v'_{i,j}$'s in the same way as CLRP (lines 1 and 7). The main difference between the two algorithms is that in each iteration, CHP calculates the noise variance v' for new query q', as well as the total privacy cost ϵ_{hybrid} for the entire plan (lines 4-6, 9-10). The final result of CHP is the parameters that minimize the budget over all iterations (line 11). Similar to algorithm CLRP, the computational cost of CHP is linear to the cardinality of Q.

```
*CHP* ($Q$, $v_{i,j}$ for all $q_{i,j}$, $q$, $v$) // Create a Hybrid Plan
// Input: $Q$, $v_{i,j}$: set of previous queries and their result noisy variance
//        $q$, $v$: incoming query and its required result variance
// Output: $v'_{i,j}$,$v'$: refined noise variance for the queries in $Q$ and for $q'$
1.  Perform lines 1-3 of algorithm CLRP
2.  Initialize $\epsilon_{min} = +\infty$
3.  Repeat
4.      Solve $v'$ from Equation (15)
5.      Compute $\epsilon = \epsilon_{hybrid}$ according to Equation (16)
6.      If $\epsilon < \epsilon_{min}$, set $\epsilon_{min}$ to $\epsilon$, and save the plan parameters
7.      Same as lines 5-15 of algorithm CLRP
8.  Until Equation (10) is satisfied
9.  Set $v' = +\infty$, and compute $\epsilon = \epsilon_{LR}$ according to Equation (11)
10. If $\epsilon < \epsilon_{min}$, set $\epsilon_{min}$ to $\epsilon$, and save the plan parameters
11. Return the set of $v'_{i,j}$ ($1 \le i \le n$, $1 \le j < m_i$) and $v'$ that achieve $\epsilon_{min}$
```

Figure 7: CHP algorithm: Create a Hybrid Plan

Note that the first iteration of CHP computes the best parameters of a TB plan, whereas the last one (lines 9-10) obtains the best LR plan. Hence, the hybrid plan subsumes the previous two plan types. Table II demonstrates an example of CHP, with parameters v=1, n=2, m_1=m_2=2, $v_{1,1}$=3, $v_{1,2}$= $v_{2,1}$=2, $v_{2,2}$=1. In each iteration, the table shows the refined variance v' for each past query q', and the plans' total privacy budget cost. In iteration 1, $v'_{i,j}$=$v_{i,j}$ for all pairs i, j. In iteration 2, CHP reduces the value of $v'_{2,1}$, which has the higher score according to Equation (17), whose new value is underlined in the table. In the next two iterations, CHP refines first $v'_{1,1}$, then $v'_{1,1}$ and $v'_{1,2}$. Finally, CHP reports minimum ϵ_{hybrid}=0.97, obtained in iteration 1, along with the corresponding plan parameters.

Table II
EXAMPLE OF HYBRID PLAN CREATION

Iteration	$v'_{1,1}$	$v'_{1,2}$	$v'_{2,1}$	$v'_{2,1}$	v'	ϵ_{hybrid}
1	3	2	2	1	15/7	0.97
2	3	2	$\underline{1}$	1	10/3	1.18
3	$\underline{2}$	2	1	1	4	1.30
4	$\underline{1}$	$\underline{1}$	1	1	+∞	1.43

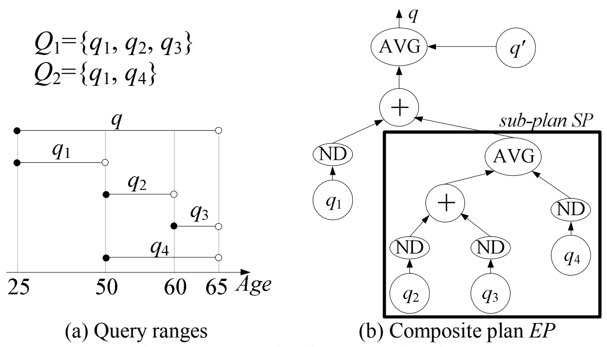
Pioneer can build a hybrid plan for queries of arbitrary sensitivity, where q is an arbitrary linear combination of existing queries (Equation (5)). Let Δ (resp. $\Delta_{i,j}$) be the sensitivity of query q (resp. $q_{i,j}$). Since q differs from q' only in its required accuracy, the sensitivity of q' is also Δ. The privacy cost of a hybrid plan is:

$$\epsilon_{hybrid} = \sqrt{\frac{2}{v'}} \cdot \Delta + \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left(\sqrt{\frac{2}{v'_{i,j}}} \cdot \Delta_{i,j} - \sqrt{\frac{2}{v_{i,j}}} \cdot \Delta_{i,j} \right). \qquad (18)$$

Meanwhile, it follows from Equation (5) that the total refined variance of queries in partition Q_i is $v'_{i,*} = \sum_{j=1}^{m_i} \left(c_{i,j}^2 \cdot v'_{i,j} \right)$. The remaining analysis and parameter computation for this general case proceeds by simply substituting this definition for $v'_{i,*}$, and Equations (18) for (16), in the hybrid plan discussion.

D. Composite Plans

Now we drop the assumption that any two different query partitions are disjoint, and handle that situation using a *composite plan*, containing multiple sub-plans. Figure 8a shows an example with four previously-answered range-counts q_1-q_4 and a new one q. The horizontal line associated with each query shows its range selection. For instance, the range of q_1 is [25-50). Clearly, $q=q_1+q_2+q_3=q_1+q_4$, so let $Q_1=\{q_1, q_2, q_3\}$ and $Q_2=\{q_1, q_4\}$. With the methods from the previous subsections, only one of Q_1, Q_2 can be used to answer q, since both contain q_1. Thus, the random noise added to the results of $q_1+q_2+q_3$ and q_1+q_4 are not independent. Consequently, if we feed both Q_1 and Q_2 to the AVG operator used in all three execution plans, the noise variance of its output becomes hard to compute, and our previous analysis, e.g., Equations (7), (15), no longer hold.



(a) Query ranges (b) Composite plan *EP*

Figure 8: Example of a composite plan

On the other hand, in the example, the *difference* between q and q_1 can be expressed in two independent linear combinations, i.e., $q-q_1=q_2+q_3=q_4$. This means that $q-q_1$ can be answered with a hybrid plan with query partitions $\{q_2, q_3\}$, $\{q_4\}$. Figure 8b displays a plan *EP* based on this idea, which contains an LR sub-span *SP* that answers $q-q_1$ using query sets $\{q_2, q_3\}$, $\{q_4\}$. Meanwhile, the backbone of *EP* is a hybrid plan, with *SP* (involving queries q_2-q_4) as one input and q_1 as the other, and independent, input. The hybrid plan here differs slightly from the one shown in Figure 6, in that the former does not have an ND operator on the *SP* input. This is because the ND operator can only take a simple query as input, not the results of a sub-plan, as explained in Section II-B.

Figure 9 illustrates the general structure of a composite plan, which resembles the TB plan in Figure 3, except that each input in the composite plan is the output of a sub-plan $SP_{i,j}$. In particular, each sub-plan can be a single query (possibly refined by an ND operator); a simple TB, LR, or hybrid plan; or (recursively) a composite plan. For instance, in Figure 8b, we have n=1, $SP_{1,1}=q_1$, and $SP_{1,2}=SP$.

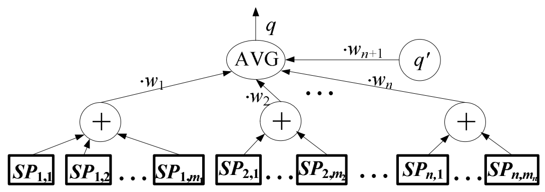


Figure 9: Structure of a composite plan

It remains to clarify the computation of (i) the best parameters for a given composite plan structure, and (ii) the construction of the best plan from the set of past queries. We first focus on the former. Let v, v', and $v_{i,j}^{SP}$ be the noise variance of incoming query q, query q', and the output of sub-plan $SP_{i,j}$, respectively. Similar to our previous analysis, the best weights in the topmost AVG operator lead to the following equation.

$$v = \frac{1}{(1/v') + \sum_{i=1}^{n}\left(1/v_{i,*}^{SP}\right)}, \text{where } v_{i,*}^{SP} = \sum_{j=1}^{m_i} v_{i,j}^{SP} \quad (19)$$

Each $v_{i,j}^{SP}$ is also recursively calculated with its inputs, using a similar analysis. For instance, in the composite plan shown in Figure 8b, let v'_1-v'_4 be the refined variance of queries q_1-q_4, respectively, and v_{SP} be the output variance of *SP*; we have $v=1/(1/(v'_1+v_{SP})+1/v')$, where $v_{SP}=1/(1/(v'_2+v'_3)+1/v'_4)$.

The privacy budget cost of the composite plan is the sum of the costs for answering new query q' and for each of the sub-plans. The following theorem states that in the best composite plan, there is no new query in a sub-plan.

Theorem 1: The best composite plan issues at most one new query q', which feeds into the top-level AVG operator.

Proof (sketch): For any composite plan *EP* in which a sub-plan *SP* issues a new query q_s, we prove that a better plan *EP'* exists with lower privacy budget cost. According to the composite plan structure, the parent operator of SP is a sum, denoted by op_{SUM}, and the parent of op_{SUM} is an AVG operator, denoted by op_{AVG}. *EP'* is identical to *EP*, except that *EP'* issues a new query q_t instead of q_s, and q_t feeds into op_{AVG}. Let v_{SP} and v'_{SP} be the noise variance of *SP* in *EP* and *EP'*, respectively, and v_s be the noise variance of q_s. Similar to the analysis of the TB plan, we obtain:

$$v_{SP} = \frac{1}{(1/v'_{SP}) + (1/v_s)}. \quad (20)$$

Let v_{SUM}, v'_{SUM} be the noise variance of op_{SUM} in *EP* and *EP'*, respectively, and v_t be the noise variance of q_t. Because the two plans differ only in *SP*, op_{AVG} has the same output variance in *EP* and *EP'*. Therefore, we have:

$$v_{SUM} = \frac{1}{(1/v'_{SUM}) + (1/v_t)}. \quad (21)$$

Combining Equations (20), (21), and the fact that $v_{SUM}=v'_{SUM}-v'_{SP}+v_{SP}$, we obtain:

$$v'_{SUM} - \frac{1}{(1/v'_{SUM}) + (1/v_t)} = v'_{SP} - \frac{1}{(1/v'_{SP}) + (1/v_s)}. \quad (22)$$

Because $v'_{SUM}>v'_{SP}$, it follows from Equation (22) that $v_s<v_t$. Thus, the privacy cost of *EP'* is lower that than of *EP*.

Let ϵ_{comp} be the total privacy budget of the composite plan, v_i (resp. v'_i) the original (resp. refined) variance of the i-th query, and v' the variance of q'. According to Lemma 3, we have:

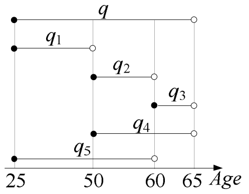
$$\epsilon_{comp} = \sum_{i=1}^{|Q|}\left(\sqrt{2/v'_i} - \sqrt{2/v_i}\right) + \sqrt{2/v'}. \quad (23)$$

Based on Equations (19) and (23), we derive the marginal accuracy benefit and privacy cost for decreasing v'_i:

$$\forall v_i, \frac{\partial v}{\partial v_i'} = \left(\frac{v}{v_{x,*}^{SP}}\right)^2 \cdot \frac{\partial v_{x,y}^{SP}}{\partial v_i'} \text{ and } \frac{\partial \epsilon_{comp}}{\partial v_i'} = \frac{-1}{v_i' \cdot \sqrt{2v_i'}}, \quad (24)$$

where $v_{x,y}^{SP}$ is the top-level sub-plan that contains q_i in its sub-tree, and $v_{x,*}^{SP}$ is the corresponding top-level sum that contains $v_{x,y}^{SP}$. The score of q_i is then defined as the ratio between its marginal accuracy benefit and marginal privacy cost. The parameter computation algorithm for the composite plan follows the same idea as for a hybrid plan (Figure 7).

Now we explain how to build a composite plan from a set of past queries Q. Suppose there are t different (but not necessarily disjoint) subsets $Q_1, \ldots, Q_t \subseteq Q$, such that the sum of the exact results in each Q_i ($1 \leq i \leq t$) equals the exact result of q. It is not always possible to use all Q_i's in computing q. Figure 10 shows an example with 5 queries and 3 sets Q_1-Q_3 containing queries that sum up to q. No composite plan can utilize all five past queries; instead, we have to discard one query subset, and form a plan with the remaining two.



$$Q_1 = \{q_1, q_2, q_3\}$$
$$Q_2 = \{q_1, q_4\}$$
$$Q_3 = \{q_3, q_5\}$$

Figure 10: Example of the selection of past query sets

Selecting query subsets that minimize the privacy budget usage for processing q is hard, since there are an exponential number of query set combinations. Figure 11 describes an effective heuristic algorithm CCP, which starts by sorting the query combinations according to their total variance (lines 1 and 2). Then, it initializes a query plan EP to answer q with the sum of queries in Q_1 (i.e., the one with the smallest total variance), and incrementally adds new query combinations to EP, in increasing order of their total variance (lines 4-9). For each subset Q_i, CCP checks whether Q_i can be merged into EP to form a new composite plan. If not, Q_i is discarded. Specifically, let $EP \backslash Q_i$ be the set of queries in EP, but not in Q_i; to merge EP and Q_i, all queries in $EP \backslash Q_i$ must be leaf nodes in EP in one simple plan. We then create a new sub-plan in EP that averages the sum of $EP \backslash Q_i$, and that of $Q_i \backslash EP$.

```
CCP ($Q$, $v_1$-$v_{|Q|}$, $q$, $v$, $Q_1$-$Q_t$) // Create a Composite Plan
// Input: $Q$, $v_1$-$v_{|Q|}$: set of previous queries and their variance
//        $q$, $v$: incoming query and its required variance
//        $Q_1$-$Q_t$: subsets of $Q$ with queries that sum up to $q$
// Output: $EP$: composite plan for answering $q$ with queries in $Q$
1. Compute the total variance for each subset $Q_i$, $1 \leq i \leq t$
2. Sort the subsets in increasing order of their total variance
3. Initialize composite plan $EP$ with queries in $Q_1$
4. For $i$=2 to $t$
5.     If no query in $Q_i$ is contained in $EP$, add $Q_i$ to EP
6.     Else
7.         If $Q_i$ and $EP$ can be merged into a composite plan
8.             Merge $Q_i$ into $EP$
9.         Else discard $Q_i$
10. Return $EP$
```

Figure 11: CCP algorithm: Create a Composite Plan

In the example of Figure 10, suppose that Q_1-Q_3 are already sorted by total variance. We initialize EP with Q_1. Then, we merge EP with Q_2 by creating a new sub-plan that averages q_2+q_3 (which are in EP but not in Q_2), and q_4 ($Q_2 \backslash EP$), which leads to the plan in Figure 8b. Finally, Q_3 and EP cannot merged, as $EP \backslash Q_3 = \{q_1, q_2, q_4\}$ are scattered across two sub-plans. Consequently, Q_3 is discarded, and CCP returns the current EP.

IV. Selection of Past Queries

This section deals with the selection of past query set Q to use to build an execution plan for a new query q. In the following, we assume all past queries satisfy the requirements mentioned in Section III: that they operate on the same dataset as q, contain the same aggregate as q, and zero-mean, Laplace noise is injected into their results as noise. Queries that do not satisfy these conditions are simply ignored. Recall that Q is divided into subsets, $Q_1, \ldots, Q_n$, each of which contains queries with a linear combination equivalent to q. A brute force approach to computing Q is to enumerate every possible combination of past queries, and verify whether they can be linearly transformed to answer q. This method incurs high computational cost. More importantly, direct combinations of existing queries that can answer q may be rare, and may not exist at all, especially when the query selections are performed on a domain with many values or high dimensionality.

Instead, Pioneer employs a novel approach that leverages special properties of ϵ-DP query processing. This technique takes linear time to the number of past queries; further, it ensures that all past queries can potentially be used. Before describing how to pick Q, we first explain how a past query q_i can contribute to the processing of q. Assume that the required noise variance of q_i and q is v_i and v, respectively. Figure 12 illustrates all possible relationships between the selection conditions of q_i and q, i.e., disjoint, overlapping, or one containing the other. (Although we use one dimensional range selections for ease of presentation, the concepts and methods described here apply to *any* selection conditions). When q_i and q are disjoint (Figure 12a), it may seem that q_i is unrelated to q. However, according to properties of ϵ-DP [6][16], answering q_i alone consumes the same amount of privacy budget as processing both q_i and another query q_x=q with accuracy requirement v_i. Accordingly, Pioneer generates such a new query q_x without consuming any privacy budget, and adds q_x to Q as a singleton.

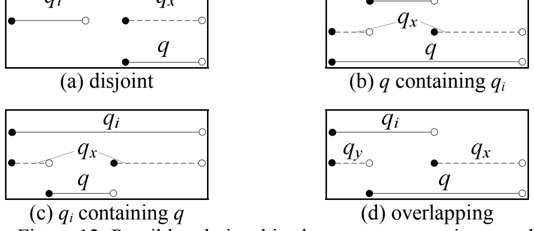
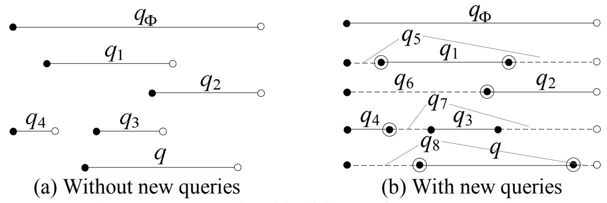


Figure 12: Possible relationships between two queries q_i and q

Likewise, when q contains q_i (Figure 12b), Pioneer creates a new query q_x (without privacy cost) with selection $q \backslash q_i$ and required variance v_i, and adds subset $\{q_i, q_x\}$ to Q. During plan computation, refining q_i with an ND operator automatically refines q_x to the same accuracy (and vice versa), without additional privacy budget usage. Therefore, the marginal accuracy benefit of refining q_i (or q_x) is doubled during plan creation. When q_i contains q (Figure 12c), it is no longer possible to issue a "free" query that answers q either directly or in combination with q_i. On the other hand, recall that all execution plans for q, except for the LR plan, issue a query $q'=q$ with required variance $v'>v$. For such plans, Pioneer additionally processes a query q_x with selection $q_i \backslash q$ and required variance v'. Answering q' and q_x consumes the same privacy budget as processing q' alone. Meanwhile, since the linear combination $q_i - q_x$ answers q, Pioneer adds $\{q_i, q_x\}$ to Q. Note that the coefficient associated with a query in Q can be negative in Equation (5). Unlike other queries, q_x cannot be refined with an ND operator. Further, the required variance of q_x, i.e., v', is unknown beforehand. To handle this, the only (minor) change in TB plan computation is that Equation (8) now involves v' on both sides of the equation, and Pioneer solves for v' there instead of obtaining v' directly from its right hand side. Similar changes apply to the computation of the hybrid and composite plans as well. Figure 12d shows the situation where q_i overlaps q, but neither contains the other. To make use of q_i, Pioneer creates two additional queries, $q_x=(q \cup q_i)\backslash q_i$ with required variance v_i, and $q_y=(q \cup q_i)\backslash q$ with required variance v', and inserts a new subset $\{q_i, q_x, q_y\}$ into Q, which answers q by $q=q_i+q_x-q_y$. As in the above discussions, the effectiveness of refining q_i is doubled as it also improves the accuracy of q_x; q_y cannot be refined, and its required variance v' is solved for during plan creation.

Next we explain how to build Q from a given set of past queries Φ. Figure 13a illustrates an example with $\Phi=\{q_1\text{-}q_4\}$, with required accuracy $v_1\text{-}v_4$. Among them, q_3 and q_4 have the same required accuracy $v_3=v_4$; further, they can share the same privacy budget, which happens, e.g., when q_4 is issued after q_3, and answered without additional privacy budget usage. Although q_4 is disjoint with q, we cannot handle it as in Figure 12a, because q_4 shares the privacy budget with q_3. Instead, Pioneer considers q_3 and q_4 as a single query that overlaps with q. To build Q, Pioneer first builds the query q_Φ whose selection clause is the union of those of $q_1\text{-}q_4$, as shown in Figure 13a. In the following, we simply say that q_Φ is the union of $q_1\text{-}q_4$. Then, Pioneer creates new queries $q_5\text{-}q_8$ (Figure 13b), with selections $q_5=q_\Phi\backslash q_1$, $q_6=q_\Phi\backslash q_2$, $q_7=q_\Phi\backslash q_3\backslash q_4$, $q_8=q_\Phi\backslash q$, respectively, and required accuracy $v_5=v_1$, $v_6=v_2$, $v_7=v_3=v_4$, $v_8=v'$, where v' is the required variance for q' in the execution plans of Section III. After that, Pioneer adds 3 subsets $\{q_1, q_5, q_8\}$, $\{q_2, q_6, q_8\}$ and $\{q_3, q_4, q_7, q_8\}$ to Q. Since q_8 is present in all three subsets, q needs a composite plan.



(a) Without new queries (b) With new queries
Figure 13: Example of building Q from past queries

Using the above method, for an incoming query q, all past queries can be used to build an execution plan for q. Pioneer does not take this option, because (i) it creates numerous new queries that share the same privacy budget with past ones, which render them less versatile in answering future queries, and (ii) the additional reduction in privacy budget usage for q gradually diminishes with increasing number of subsets in Q. Instead, Pioneer chooses the k (a system parameter) groups of past queries with lowest total variance to form the set Φ. Each group of queries share the same privacy budget, e.g., q_3 and q_4 in Figure 13. Figure 14 shows the algorithm *CQ* for building Q, for the set of queries Φ. CQ builds the union q_Φ of all queries in Φ, and creates a new query for each group, as well as q, that extends their respective unions to q_Φ (lines 5-6). For instance, in Figure 13, the new query q_7 created for the $\{q_3, q_4\}$ is the difference between q_Φ and the union of q_3 and q_4. Finally, CQ forms a subset for each group, and adds it to Q (lines 7 and 8).

```
CQ ($\Phi$, $k$, $v_1$-$v_k$, $q$, $v$) // Create $Q$
// Input: $\Phi$: selected subset of previous queries
//        $k$: target number of groups in $\Phi$
//        $v_1$-$v_k$: result variance of the $i$-th group of queries in $\Phi$
//        $q$, $v$: incoming query and its required variance
// Output: $Q$: set of queries used in the execution plan of $q$
1. Build the union $q_\Phi$ of all queries in $\Phi$
2. Create new query $q_n$ with selection $q_\Phi\backslash q$ and required variance $v'$
3. Initialize $Q$ to empty
4. For $i$=1 to $k$
5.     Build the union $q_u$ of all queries in the $i$-th group of $\Phi$
6.     Create query $q_g$ with selection $q_\Phi\backslash q_u$ and required variance $v_i$
7.     Form a subset $Q_i$ consisting of the $i$-th query group, $q_n$ and $q_g$
8.     Add $Q_i$ to $Q$
9. Return $Q$
```
Figure 14: CQ algorithm: Create Q

Finally, we mention that query processing in existing DP-compliant DBMSs (e.g., PINQ [16]) can be improved using ideas from this section. We call the new method *Baseline*, and evaluate it in our experiments. Specifically, when a new query q arrives, Baseline checks whether there exists a past query q_i whose selection is disjoint with that of q (Figure 12a). If so, the method *coalesces* q_i and q into a single query, e.g., using the partition operator [16]. The privacy budget usage of the coalesced query is equal to that of whichever of q_i and q has the higher accuracy requirement. This approach consumes less privacy budget than processing q_i and q separately.

V. EXPERIMENTS

We implemented Pioneer, Baseline (described in Section IV), and a naive algorithm (*Naive*) that processes each query independently. Naive is used in most existing systems such as PINQ [16]. All methods are evaluated on a 2.5 GHz Intel Core i5 CPU with 4GBytes of RAM. We exclude offline methods in our comparisons since they are ineffective for queries with different accuracy requirements, as explained in Section II-B. The experiments use mainly three performance metrics: total privacy budget usage for answering a sequence of queries, running time for plan and noise computation, and the number of queries each method can process under 1-DP. Properties of

DP ensure that the evaluation results only depend on the query workload, *not* the underlying database. To test Pioneer on real datasets, at the end of this section we also evaluate its average relative error for a series of queries, each with a given privacy budget ϵ. To do this, we use Pioneer to compute the query accuracy that leads to budget usage ϵ through binary search. The real dataset used is *Social Network* [10], which contains the node degrees of a social network with 11,342 nodes.

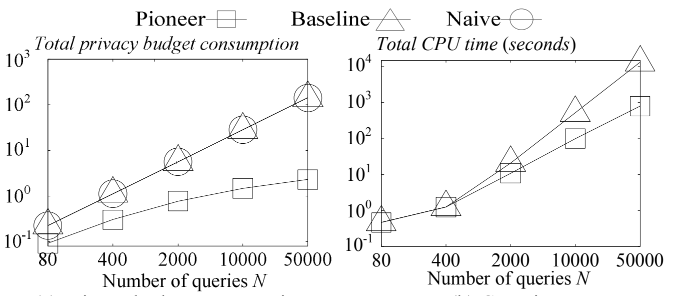
As there are no standard workloads for ϵ-DP, we generated simple synthetic ones, as follows. Each query is a 1D range-count, whose sensitivity is 1. The range selection is over a finite, discrete domain. The range width, start position, and required accuracy (noise variance) of each query follow Gaussian distributions, whose mean and standard deviation are parameters under investigation. We chose Gaussian since it models well many variables in practice, e.g., the age of patients suffering from a certain disease. Further, a high standard deviation (STD) approximates uniform distribution, where as a low STD models high skewness. We investigate the impact of 7 workload parameters: number of queries N, size of the queried domain D, the mean l_{avg} and STD l_{std} of the query range width, the mean v_{avg} and STD v_{std} of the required noise variance for each query, and the STD s_{std} of the range start position. We also evaluate the impact of parameter k, which determines the number of past queries used in a plan (Section IV). Table III lists the parameter values, with defaults in bold.

Table III
PARAMETERS INVESTIGATED IN THE EXPERIMENTS

Parameter	Range and (in bold) Default
N	80, 400, 2000, **10000**, 50000
D	500, 1000, 2000, 5000, **10000**, 20000
l_{avg}	20, 80, **320**, 1280, 5120
l_{std}	2, **10**, 50, 250, 1250
v_{avg}	10000, 50000, **250000**, 1250000, 6250000
v_{std}	1000, 5000, **25000**, 100000, 500000
s_{std}	**10**, 20, 50, 100, 200
k	0, 1, 2, 3, **4**, 8, 32, 64, 256

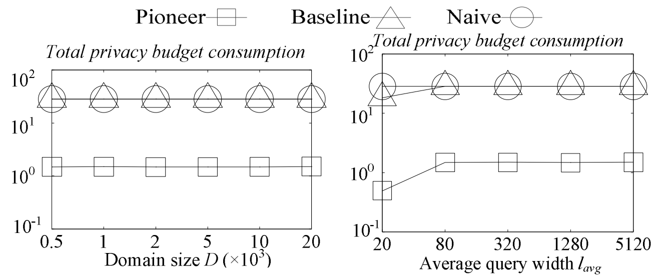
In each experiment, we vary one parameter, and fix the rest to their defaults. We ran each configuration 10 times, and report the average results. We use relatively large noise variance (e.g., 250,000, corresponding to standard deviation 500), which may suit applications with relatively large values for the actual results (e.g., population counts). Nevertheless, the magnitude of the noise variance does not affect the relative performance of the methods, or the trends in their performance with varying parameters. For instance, if we decrease v_{avg} from 250,000 to 2,500, the privacy budget cost of each method will simply multiply by a factor of 10 in each setting.

Figure 15a plots the total privacy budget consumption as a function of the workload size N. Both axes are in logarithmic scale. The total budgets of all methods increase with N, with Pioneer consistently outperforming the other two, often by more than an order of magnitude for a relatively large workload. Meanwhile, the budget consumption of Naive and Baseline grow linearly with N, whereas the privacy cost of Pioneer increases at a much slower pace and eventually flattens. This is because the more queries are processed, the higher the chance for Pioneer to identify past queries that answer an incoming query with sufficient accuracy, leading to little budget usage. Figure 15b shows the CPU time for Pioneer and Baseline for the entire workload. The CPU time for NAIVE is omitted, as it lies flat along the X axis. Pioneer needs no more than a few milliseconds to optimize a query, which is negligible, especially considering its vast reduction in privacy budget consumption. Hence, in the following experiments, we omit CPU time results.

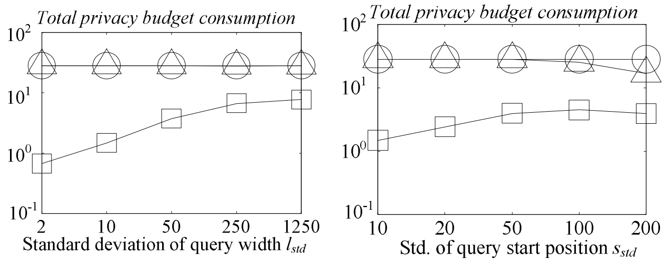


(a) Privacy budget consumption (b) CPU time
Figure 15: Effect of the number of queries

Figure 16a varies the size D of the domain on which the range selections are performed. The results suggest that D does not have a significant impact on privacy cost. This is true even for very small domains (e.g., 500), where the query (with default average length 320) covers a large portion of the domain. Hence, unlike query optimization in traditional DBMSs, selectivity may not be a critical factor in our application. Similarly, Figure 16b investigates the effect of another parameter that affects selectivity, namely the average width l_{avg} of the query range. The performance of all three methods remains stable, except for very small values of l_{avg}, e.g., 20, which are comparable to the standard deviation l_{std} of the query width. Such a parameter combination generates queries with tiny ranges, which tend to be disjoint, and, thus, can often be coalesced by Baseline and Pioneer, leading to lower privacy costs. The privacy budget usage of Naïve however is unaffected by l_{avg}, since it processes each query independently.



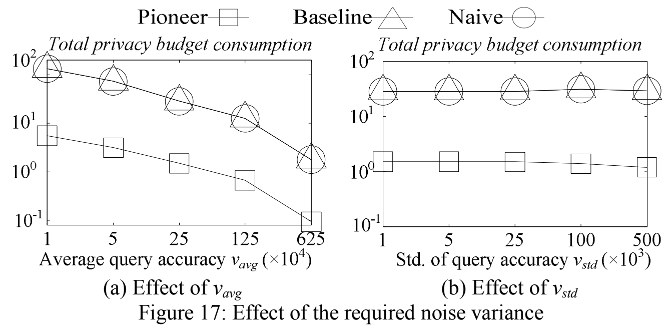
(a) Effect of D (b) Effect of s_{std}



(c) Effect of l_{avg} (d) Effect of l_{std}
Figure 16: Effect of query characteristics

Figure 16c compares the methods for different values of the standard deviation of the range width l_{std}. Naive and Baseline are not sensitive to l_{std}, but the privacy budget usage of Pioneer grows with l_{std}. The reason is that with small values of l_{std}, queries tend to be rather similar, sometimes even identical. For these settings, LR plans and hybrid/composite plans with lots of query refinement perform quite well, since they often lead to a small number of repeatedly refined queries with very high accuracy. The combinations of these queries often answer a considerable fraction of new queries without using additional privacy budget, which reduces total privacy costs. As l_{std} grows, this phenomenon gradually diminishes, and the performance of Pioneer stabilizes. Nevertheless, in terms of privacy budget usage, Pioneer consistently outperforms the other two methods, even for large values of l_{std}. Figure 16d shows the effect of the standard deviation of the start position of query ranges s_{std}. The mean value for query start positions does not affect our evaluation results. Pioneer is again the winner in all settings. As s_{std} grows, the privacy costs of Pioneer increase at first, reaching a peak at around s_{std}=50, and drop slightly after that. The initial privacy cost increase is due to the fact that queries become more diverse, reducing the effectiveness of LR-like plans, as explained above. The subsequent drop is caused by more disjoint queries being coalesced together, which also benefits the Baseline method.

Next we focus on the impact of the required query accuracy. Figure 17a shows the results with different average required variance v_{avg}. As expected, the total privacy budget usage decreases with growing v_{avg}, and the trend agrees with Equation (3). The standard deviation v_{std} of required noise variance has a subtle effect, as show in Figure 17b: when v_{std} becomes high (e.g., when $v_{std} \geq v_{avg}$=250,000), the budget consumption slightly decreases. This is because such settings generate queries with very different accuracy requirements, and the most accurate ones can often be composed to answer the most inaccurate ones with little or no budget consumption.



(a) Effect of v_{avg} (b) Effect of v_{std}

Figure 17: Effect of the required noise variance

To demonstrate the practical utility of Pioneer, we repeated all experiments, and report the number of queries answered by a DP-compliant DBMS with a total privacy budget ϵ=1, which is a popular value for ϵ. The maximum number of queries is bounded by the size of the workload N, which is 10,000 by default. Figure 18 show the results of varying parameters l_{avg}, l_{std}, v_{avg}, and v_{std}. The comparisons of Pioneer, Baseline and Naive lead to the same conclusions as before. In Figure 18d, the effect of varying v_{std} on Pioneer can be more clearly observed. In all settings, Pioneer handles 5-10 times as many queries as Baseline and Naive.

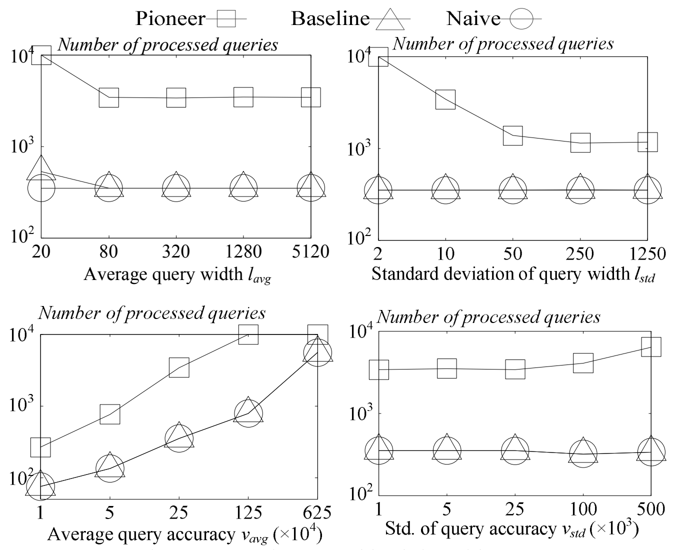
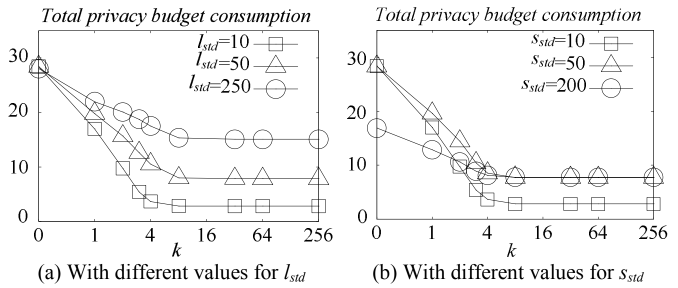


Figure 18: Maximum workload size with 1-DP

Having established the superiority of Pioneer over the other methods, we next investigate the intrinsic properties of Pioneer. Figure 19 shows the effect of parameter k, i.e., the number of (possibly coalesced) previous queries used to answer new query q. Since there is no competitor, we show the impact of k with different values of l_{std} (standard deviation of range width) and s_{std} (standard deviation of query start position). For all tested values of l_{std} and s_{std}, Pioneer's privacy budget usage decreases quickly as k grows, until reaching a stable point at around k=4. After that, further increases in k no longer have a significant impact. The reason for this behavior is that larger k values cause more previous queries to be involved in the plan. The initial few chosen by our query selection algorithm are those that represent the new query q most accurately, leading to a sharp privacy cost decline. After that, adding more queries with lower accuracy does not help much, according to Equation (15). Meanwhile, a high value for k leads to a complicated plan, which incurs higher computational costs. Overall, values of k from 4 to 10 strike a good balance.



(a) With different values for l_{std} (b) With different values for s_{std}

Figure 19: Effect of parameter k on privacy budget usage

Finally, Figure 20 evaluates the methods on a real dataset *Social Networks*, and report the average relative error, i.e., the ratio between noise and exact query answer, for a query sequence where each query has a given privacy budget ϵ=0.001. The main difference from the results above is that certain parameters (e.g., query length) affect the exact query answer, and, thus, the relative error. Nevertheless, Pioneer is again the clear winner in all settings.

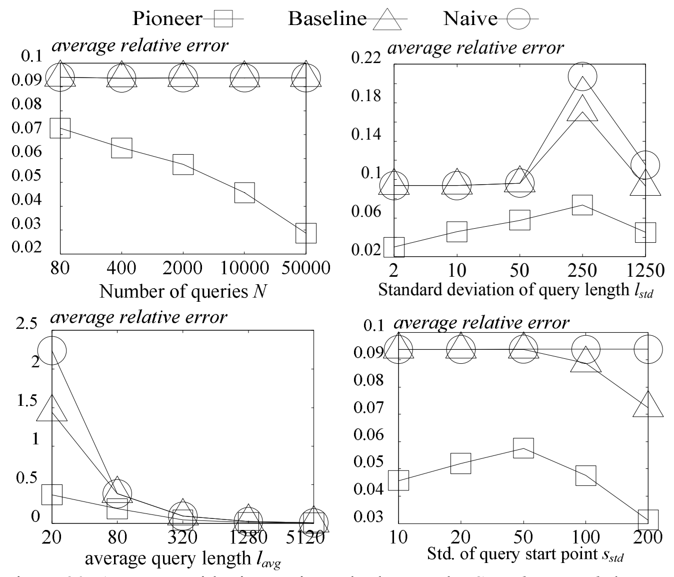


Figure 20: Accuracy with given privacy budget on the *Social Network* dataset

Summarizing the experiments, Pioneer outperforms Baseline and Naive in all settings in terms of both privacy budget consumption, and the maximum number of queries that can be processed under 1-DP. There is a considerable performance gap between Pioneer and the other two, often over an order of magnitude. CPU costs for plan selection are negligible in most settings. Thus, we believe that Pioneer is the method of choice for query optimization in a DP-compliant DBMS.

VI. CONCLUSION

This paper proposed Pioneer, an effective and efficient query optimizer for DP-compliant DBMSs. Pioneer builds an execution plan that reuses past queries, in an effort to minimize privacy budget usage. It also includes an effective algorithm for selecting past queries that contribute to answering the incoming one, and both plan computation and past query selection incur negligible computational costs. Extensive experiments show that for a given privacy budget, Pioneer can process 5-10 times as many range-count queries as the naive solution commonly used in existing systems. For a fixed workload of range queries, Pioneer typically consumes an order of magnitude less privacy budget than existing methods. The current version of Pioneer focuses on minimizing the privacy budget usage only for the current query q under processing. Hence, one option for future work is to select the query plan based on both the privacy cost of q, and the plan's expected impact for future queries.

ACKNOWLEDGEMENT

Shangfu Peng did his work while doing an internship at ADSC. This work is by Singapore's A*STAR, under SERC Grant 102-158-0074.

VII. REFERENCES

[1] B. Barak, K. Chaudhuri, C. Dwork, S. Kale, F. McSherry, K. Talwar. Privacy, accuracy, and consistency too: a holistic solution to contingency table release. *PODS*, 2007.
[2] R. Chen, N. Mohammed, B. Fung, B. Desai, L. Xiong. Publishing set-valued data via differential privacy. *VLDB*, 2011.
[3] G. Cormode, M. Procopiuc, E. Shen, D. Srivastava, T. Yu. Differentially private spatial decompositions. *ICDE*, 2012.
[4] C. Dwork. Differential privacy. *ICALP*, 2006.
[5] C. Dwork, K. Kenthapadi, F. McSherry, I. Mironov, M. Naor. Our data, ourselves: privacy via distributed noise generation. *EUROCRYPT*, 2006.
[6] C. Dwork, F. McSherry, K. Nissim, A. Smith. Calibrating noise to sensitivity in private data analysis. *TCC*, 2006.
[7] B. Ding, M. Winslett, J. Han, Z. Li. Differentially private data cubes: optimizing noise sources and consistency. *SIGMOD*, 2011.
[8] A. Friedman, A. Schuster. Data mining with differential privacy. *KDD*, 2010.
[9] M. Hardt, G. Rothblum. A multiplicative weights mechanism for privacy-preserving data analysis. *FOCS*, 2010.
[10] M. Hay, V. Rastogi, G. Miklau, D. Suciu. Boosting the accuracy of differentially private histograms through consistency. *VLDB*, 2010.
[11] D. Kifer, A. Machanavajjhala. No free lunch in data privacy. *SIGMOD*, 2011.
[12] D. Kifer, A. Machanavajjhala. A Rigorous and customizable framework for privacy. *PODS*, 2012.
[13] C. Li, M. Hay, V. Rastogi, G. Miklau, A. McGregor. Optimizing linear counting queries under differential privacy. *PODS*, 2010.
[14] C. Li, G. Miklau. An adaptive mechanism for accurate query answering under differential privacy. *PVLDB*, 5(6), 2012.
[15] Y. Li, Z. Zhang, M. Winslett, Y. Yang. Compressive mechanism: utilizing sparse representation in differential privacy. *WPES*, 2011.
[16] F. McSherry. Privacy integrated queries. *SIGMOD*, 2009.
[17] A. Machanavajjhala, D. Kifer, J. Abowd, J. Gehrke, L. Vilhuber. Privacy: theory meets practice on the map. *ICDE*, 2008.
[18] A. Machanavajjhala, A. Korolova, A. Sarma. Personalized social recommendations: accurate or private? *PVLDB*, 4(7), 2011.
[19] F. McSherry, R. Mahajan. Differentially-private network trace analysis. *SIGCOMM*, 2010.
[20] F. McSherry, K. Talwar. Mechanism design via differential privacy. *FOCS*, 2007.
[21] P. Mohan, A. Thakurta, E. Shi, D. Song, D. Culler. GUPT: privacy preserving data analysis made easy. *SIGMOD*, 2012.
[22] S. Peng, Y. Yang, Z. Zhang, M. Winslett, Y. Yu. DP-tree: indexing multi-dimensional data under differential privacy. *SIGMOD*, 2012, poster.
[23] V. Rastogi, S. Nath. Differentially private aggregation of distributed time-series with transformation and encryption. *SIGMOD*, 2010.
[24] A. Roth, T. Roughgarden. Interactive privacy via the median mechanism. *STOC*, 2010.
[25] X. Xiao, G. Bender, M. Hay, J. Gehrke. iReduct: differential privacy with reduced relative errors. *SIGMOD*, 2011.
[26] X. Xiao, G. Wang, J. Gehrke. Differential privacy via wavelet transforms. *ICDE*, 2010.
[27] J. Xu, Z. Zhang, X. Xiao, Y. Yang, G. Yu. Differentially private histogram publication. *ICDE*, 2012.
[28] Y. Yang, Z. Zhang, G. Miklau, M. Winslett, X, Xiao. Differentially private data publication and analysis. *SIGMOD*, 2012, tutorial.
[29] G. Yuan, Z. Zhang, M. Winslett, X. Xiao, Y. Yang, Z. Hao. Low-rank mechanism: optimizing batch queries under differential privacy. *PVLDB*, 5(11), 2012.
[30] J. Zhang, Z. Zhang, X. Xiao, Y. Yang, M. Winslett, Functional mechanism: regression analysis under differential privacy. *PVLDB*, 5(11), 2012.